

PHYS-1200 PHYSICS II

HOMEWORK #22 SOLUTIONS

19. Let N be the number of atoms per unit volume and n be the number of free electrons per unit volume. Then, the number of free electrons per atom is n/N . We use the result of Exercise 11 to find n : $E_F = An^{2/3}$, where $A = 3.65 \times 10^{-19} \text{ m}^2 \cdot \text{eV}$. Thus,

$$n = \left(\frac{E_F}{A} \right)^{3/2} = \left(\frac{11.6 \text{ eV}}{3.65 \times 10^{-19} \text{ m}^2 \cdot \text{eV}} \right)^{3/2} = 1.79 \times 10^{29} \text{ m}^{-3} .$$

If M is the mass of a single aluminum atom and d is the mass density of aluminum, then $N = d/M$. Now,

$$M = (27.0 \text{ g/mol}) / (6.022 \times 10^{23} \text{ mol}^{-1}) = 4.48 \times 10^{-23} \text{ g},$$

so

$$N = (2.70 \text{ g/cm}^3) / (4.48 \times 10^{-23} \text{ g}) = 6.03 \times 10^{22} \text{ cm}^{-3} = 6.03 \times 10^{28} \text{ m}^{-3} .$$

Thus, the number of free electrons per atom is

$$\frac{n}{N} = \frac{1.79 \times 10^{29} \text{ m}^{-3}}{6.03 \times 10^{28} \text{ m}^{-3}} = 2.97 \approx 3 .$$

25. (a) Combining Eqs. 41-2, 41-3 and 41-4 leads to the conduction electrons per cubic meter in zinc:

$$n = \frac{2(7.133 \text{ g/cm}^3)}{(65.37 \text{ g/mol}) / (6.02 \times 10^{23} \text{ mol})} = 1.31 \times 10^{23} \text{ cm}^{-3} = 1.31 \times 10^{29} \text{ m}^{-3} .$$

(b) From Eq. 41-9,

$$E_F = \frac{0.121 h^2}{m_e} n^{2/3} = \frac{0.121 (6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2 (1.31 \times 10^{29} \text{ m}^{-3})^{2/3}}{(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ J/eV})} = 9.43 \text{ eV} .$$

(c) Equating the Fermi energy to $\frac{1}{2} m_e v_F^2$ we find (using the $m_e c^2$ value in Table 37-3)

$$v_F = \sqrt{\frac{2E_F c^2}{m_e c^2}} = \sqrt{\frac{2(9.43 \text{ eV})(2.998 \times 10^8 \text{ m/s})^2}{511 \times 10^3 \text{ eV}}} = 1.82 \times 10^6 \text{ m/s} .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{m_e v_F} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(1.82 \times 10^6 \text{ m/s})} = 0.40 \text{ nm} .$$

48. (a) For copper, Eq. 41-10 leads to

$$\frac{d\rho}{dT} = [\rho\alpha]_{\text{Cu}} = (2 \times 10^{-8} \Omega \cdot \text{m})(4 \times 10^{-3} \text{K}^{-1}) = 8 \times 10^{-11} \Omega \cdot \text{m} / \text{K} .$$

(b) For silicon,

$$\frac{d\rho}{dT} = [\rho\alpha]_{\text{Si}} = (3 \times 10^3 \Omega \cdot \text{m})(-70 \times 10^{-3} \text{K}^{-1}) = -2.1 \times 10^2 \Omega \cdot \text{m} / \text{K} .$$

53. We equate E_F with $\frac{1}{2}m_e v_F^2$ and write our expressions in such a way that we can make use of the electron mc^2 value found in Table 37-3:

$$v_F = \sqrt{\frac{2E_F}{m}} = c \sqrt{\frac{2E_F}{mc^2}} = (3.0 \times 10^8 \text{ m/s}) \sqrt{\frac{2(7.0 \text{ eV})}{5.11 \times 10^5 \text{ eV}}} = 1.6 \times 10^3 \text{ m/s} .$$

Q41.11 B and D