PHYS-1200 PHYSICS II

HOMEWORK #21 SOLUTIONS

2. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and the hc = 1240 eV · nm value developed in problem 83 of Chapter 38 by writing Eq. 39-4 as

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (hc)^2}{8(mc^2)L^2}.$$

The energy to be absorbed is therefore

$$\Delta E = E_4 - E_1 = \frac{(4^2 - 1^2)h^2}{8m_e L^2} = \frac{15(hc)^2}{8(m_e c^2)L^2} = \frac{15(1240\text{eV}\cdot\text{nm})^2}{8(511\times10^3\text{eV})(0.250\text{nm})^2} = 90.3\text{eV}.$$

13. The probability that the electron is found in any interval is given by $P = \int |\psi|^2 \, dx$, where the integral is over the interval. If the interval width Δx is small, the probability can be approximated by $P = |\psi|^2 \, \Delta x$, where the wave function is evaluated for the center of the interval, say. For an electron trapped in an infinite well of width L, the ground state probability density is

$$|\psi|^2 = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right),$$

so

$$P = \left(\frac{2\Delta x}{L}\right)\sin^2\left(\frac{\pi x}{L}\right).$$

(a) We take L = 100 pm, x = 25 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(25\,\text{pm})}{100\,\text{pm}}\right] = 0.050.$$

(b) We take L = 100 pm, x = 50 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(50\,\text{pm})}{100\,\text{pm}}\right] = 0.10.$$

(c) We take L = 100 pm, x = 90 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(90\,\text{pm})}{100\,\text{pm}}\right] = 0.0095.$$

14. We follow Sample Problem 39-3 in the presentation of this solution. The integration result quoted below is discussed in a little more detail in that Sample Problem. We note that the arguments of the sine functions used below are in radians.

(a) The probability of detecting the particle in the region $0 \le x \le \frac{L}{4}$ is

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_0^{\pi/4} \sin^2 y\, dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)_0^{\pi/4} = 0.091.$$

(b) As expected from symmetry,

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{\pi} \sin^2 y\, dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)_{\pi/4}^{\pi} = 0.091.$$

(c) For the region $\frac{L}{4} \le x \le \frac{3L}{4}$, we obtain

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{3\pi/4} \sin^2 y\, dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)_{\pi/4}^{3\pi/4} = 0.82$$

which we could also have gotten by subtracting the results of part (a) and (b) from 1; that is, $1 - 2(0.091) = 0.82$.

18. From Fig. 39-9, we see that the sum of the kinetic and potential energies in that particular finite well is 280 eV. The potential energy is zero in the region $0 < x < L$. If the kinetic energy of the electron is detected while it is in that region (which is the only region where this is likely to happen), we should find $K = 280$ eV.

Q41.07 much less than